Nonlinear acceleration wave propagation in the DKM theory

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Abstract

We study the evolutionary development of an acceleration wave propagating in a saturated porous material according to a Biot theory proposed by Donskoy, Khashanah and McKee. The theory is fully nonlinear, includes dissipation, and the analysis presented is exact. We derive sufficient conditions to show that two distinct waves propagate, a fast wave and a slower wave. A solution for the wave amplitude is presented for a wave moving into an equilibrium region.

keywords: acceleration waves; porous media; dissipation

1 Introduction

The subject of nonlinear acoustic wave propagation is one of intense recent interest as witnessed by the contributions of Christov (2016), Crighton (1979), Dazel and Tournat (2010), Donskoy et al. (1997), Jordan (2005), Jordan (2016), Pushkina (2012), Tong et al. (2017), and the many references therein.

There are various mathematical approaches to studying acoustic waves in water, elastic materials, porous media, and other materials. For example, there are weakly nonlinear expansion techniques, see e.g. Dazel and Tournat (2010), Donskoy et al. (1997), Pushkina (2012), Tong et al. (2017). In addition, there are analyses for handling acoustic travelling waves, see e.g. Christov (2016), Jordan and Saccomandi (2012), Jordan et al. (2014, 2018); and for dealing with acoustic solitary waves, see e.g. Chin-Bing et al. (2003), Chin-Bing et al. (2007), Jordan (2020), Warn-Varnas et al. (2003, 2005). The last mentioned are very important in dealing with underwater detection devices and specifically dealing with solving problems directly for solitary waves in the Yellow Sea and in the Strait of Messina. The acoustic travelling wave and acoustic solitary wave methods may also lead to interesting nonlinear mathematical models such as Kuznetsov's equation or the Westerveld equation, or the Jordan - Moore - Gibson - Thompson equation, see e.g. Crighton (1979), Jordan (2016). The Jordan - Moore - Gibson - Thompson equation is a third order in time equation which describes the nonlinear propagation of sound which avoids the paradox of an infinite speed which is associated with second order strongly damped models of nonlinear acoustics such as the Kuznetsov and Westerveld equations, see Kaltenbacher and Lasiecka (2012). A further mathematical technique for dealing with nonlinear wave propagation is that of acceleration waves, see e.g. Altenbach et al. (2010), Christov (2016), Ciarletta and Straughan (2006), Ciarletta et al. (2010), Ciarletta et al. (2013), Dolfin and Rogolino (1998), Eremeyev (2005), Eremeyev et al. (2018), Green (1963, 1964), Jordan (2005), Ostoja Starzewski and Trebicki (1999), Ostoja Starzewski and Trebicki (2006), Paoletti

(2012), Truesdell and Noll (1992), Truesdell and Toupin (1960), Ziv and Shmuel (2019). The topic of acceleration waves is particularly pertinent to the present article.

This article fosusses on wave motion in a nonlinear porous material of Biot type, cf. Biot (1962, 1972, 1973). Nonlinear wave motion in such media has been studied recently by Dazel and Tournat (2010), Donskoy et al. (1997), Pushkina (2012), Tong et al. (2017). We are particularly interested in an acceleration wave analysis. Such analysis employs no approximations and yields an exact solution to a fully nonlinear problem. To facilitate our discussion of acceleration waves in porous media we now introduce the equations of motion derived by Donskoy et al. (1997).

2 The DKM equations

The equations of Donskoy et al. (1997) are one-dimensional and introduce the solid displacement $u(x,t)$ and the fluid displacement relative to the solid $w(x,t)$, and their equations may be written as

$$\begin{aligned}\rho u_{tt}+\rho_f w_{tt}&=a_{11}u_{xx}+a_{12}w_{xx}\\&\quad-c_{11}u_xu_{xx}-c_{12}(u_xw_x)_x-d_{12}w_xw_{xx}\end{aligned}\tag{1}$$

and

$$\begin{aligned}\rho_f u_{tt}+m'w_{tt}+\frac{\eta}{\kappa}w_t&=a_{12}u_{xx}+a_{22}w_{xx}\\&\quad-c_{21}u_xu_{xx}-c_{22}(u_xw_x)_x-d_{22}w_xw_{xx}\end{aligned}\tag{2}$$

where subscript t or x denotes partial differentiation with respect to t or x, e.g. $u_t\equiv\partial u/\partial t$. The terms ρ and ρ_f are the solid and fluid densities, a_{ij} are defined by

$$a_{11}=\lambda+2\mu+\alpha^2M,\quad a_{12}=\alpha M,\quad a_{22}=M,$$

where λ and μ are the Lamé coefficient and shear modulus and α, M are elastic coefficients of Biot, see Tong et al. (2017). The constants c_{ij} and d_{ij} depend on parameters of the porous medium and are given explicitly in Donskoy et al. (1997), equations (8) and (21), although we take the representations given in Tong et al. (2017). The coefficient of the dissipation term is composed of η and κ which are the dynamic viscosity of the fluid and the permeability, respectively. While Donskoy et al. (1997) introduce the dissipation term they discard it in their wave analysis. In this work we present results both with and without dissipation.

The technique adopted in Dazel and Tournat (2010), Donskoy et al. (1997), Pushkina (2012), Tong et al. (2017), is a weakly nonlinear one whereby they attempt to obtain an approximate solution for u and w. For example, Tong et al. (2017), p. 759, *assume* u and w may be expanded as

$$u=u^{(0)}+u^{(1)}+o(u^{(1)}),\qquad w=w^{(0)}+w^{(1)}+o(w^{(1)}),\tag{3}$$

with $u^{(0)}>>u^{(1)}$ and $w^{(0)}>>w^{(1)}$, "*and the higher order terms $o(*)$ can be omitted without causing errors*". They find linear equations for $u^{(0)},w^{(0)}$ which are solved in exponentials in x and t, and at the next stage of the expansion they find equations for $u^{(1)},w^{(1)}$ which are linear in these variables but they are forced by nonlinear terms in $u^{(0)},w^{(0)}$. As might be expected, this gives rise to a different frequency for the nonlinear displacement. We stress that the procedure associated to (3) is formal and there is no serious consideration of convergence whatsoever.

To introduce the idea of an acceleration wave we refer to Truesdell and Noll (1992), pp. 267–294, who deal with an acceleration wave in the context of a nonlinear elastic body. Truesdell and Noll (1992), p. 267, write that "*a singular surface of second order with respect to the* *deformation* $x_i = x_i(X_A, t)$ *is defined as a surface across which the functions* x_i *and their first derivatives are continuous, but at least one of the second derivatives* $x^i_{,AB}$ *suffers a jump discontinuity."* They recall the kinematic conditions of compatibility for such surfaces, Truesdell and Toupin (1960), and note that $[\ddot{x}_i] = U^2 a_i$, "*where the brackets denote the jump occasioned; where the vector* **a**, *characterizing the strength of the discontinuity, is called the amplitude of the singularity and is assumed different from zero, ..., and where* U *is called the local speed of propagation. If* $U \neq 0$ *the surface propagates and is therefore called a wave; ... such a wave must carry a jump of the acceleration, it is called an acceleration wave. It is customary to identify such waves physically with sound waves."*

Before we develop an exact fully nonlinear acceleration wave analysis for the DKM equations we recall that Green (1964) showed that under appropriate conditions in an elastic body the amplitude of an acceleration wave could tend to infinity in a finite time. This is intimately connected to the formation of a shock wave and after the blow-up time the shock would evolve according to the physics of shock waves, cf. Fu and Scott (1991). We find exactly the same is true for an acceleration wave in the DKM equations for porous media, provided the amplitude satisfies certain conditions presented precisely here. It is well known that shock waves may be involved in earthquake evolution, see e.g. Arif et al. (2012), Bowden and Tsai (2017), de Arcangelis et al. (2018), Field et al. (1997), Gomberg and Johnson (2005), Griffa et al. (2013), Johnson and Jia (2005), Johnson et al. (2008), and as these papers report, the mechanisms are not fully understood. Indeed, for earthquakes and even landslides such effects as plasticity or hysteresis may need to be included, e.g. Green (1963), Loret et al. (1997), Guyer and Johnson (1999). While we are not saying acceleration wave evolution into a shock is connected to earthquakes or landslides, we do believe that an exact soultion to this evolution problem in a nonlinear theory of porous media is of interest and should not be overlooked.

We stress that an acceleration wave analysis provides exact information to a completely nonlinear problem. There is no lack of rigour as in a weakly nonlinear expansion technique. In this regard, we should point out that for some of the nonlinear acoustic models such as those of Kuznetsov, Westerveld, and Jordan - Moore - Gibson - Thompson, there are available highly rigorous analyses of solution behaviour, see Kaltenbacher and Lasiecka (2012), Kaltenbacher and Nikolić (2019), Kaltenbacher et al. (2012a), Kaltenbacher et al. (2012b).

3 Acceleration waves in the DKM theory

We consider propagation of a three-dimensional plane wave along the x-axis for equations (1) and (2). We suppose $u(x,t)$, $w(x,t)$ are C^1 everywhere in x and t, while u_{tt}, u_{tx}, u_{xx}, w_{tt}, w_{tx}, w_{xx} and their higher derivatives suffer a finite discontinuity across a surface $\mathscr{S}$. This singular surface is called an acceleration wave.

The jump of a quantity f across $\mathscr{S}$ is denoted by $[f]$ where

$$[f] = \lim_{x \to x^-} f(x,t) - \lim_{x \to x^+} f(x,t).$$

The analysis of an acceleration wave proceeds via the Hadamard relation and compatibility conditions across $\mathscr{S}$ given in Truesdell and Toupin (1960). Due to the regularity of u one may employ the Hadamard relation to see that

$$0 = \frac{\delta}{\delta t}[u_t] = [u_{tt}] + V[u_{tx}] \tag{4}$$

and

$$0 = \frac{\delta}{\delta t}[u_x] = [u_{tx}] + V[u_{xx}], \tag{5}$$

where $\delta/\delta t$ is the rate of change as seen by an observer on the wave and V is the wavespeed. From (4) and (5) one finds

$$[u_{tt}] = V^2[u_{xx}]. \tag{6}$$

To preceed, take the jumps of (1) and (2), and use (6) to obtain

$$\begin{aligned}
&(\rho V^2 - a_{11} + c_{11}u_x^+ + c_{12}w_x^+)[u_{xx}] \\
&\qquad + (\rho_f V^2 - a_{12} + c_{12}u_x^+ + d_{12}w_x^+)[w_{xx}] = 0, \\
&(\rho_f V^2 - a_{12} + c_{21}u_x^+ + c_{22}w_x^+)[u_{xx}] \\
&\qquad + (m'V^2 - a_{22} + c_{22}u_x^+ + d_{22}w_x^+)[w_{xx}] = 0.
\end{aligned} \tag{7}$$

Define the wave amplitudes A and B by

$$A = [u_{tt}], \quad B = [w_{tt}]. \tag{8}$$

Equations (7) yield the wavespeeds for an acceleration wave advancing into an arbitrary state. We now concentrate on the case where the wave moves into a saturated porous body in equilibrium so that u^+ and w^+ are constant and $u_x^+ = 0$, $w_x^+ = 0$. In this case equations (7) yield for non-zero A, B, the wavespeed equation

$$\begin{aligned}
(\rho m' - \rho_f^2)V^4 - V^2\{\rho M + \lambda m' + 2\mu m' + m'\alpha^2 M \\
- 2\rho_f\alpha M\} + (\lambda + 2\mu)M = 0.
\end{aligned} \tag{9}$$

It is noteworthy that equation (9) has the same form as equation (27) of Donskoy et al. (1997) excepting that since they sought a solution of form $\exp[i(\omega t - kx)]$ the term V in our case is equivalent to $V = \omega/k$ where ω is frequency and k is the wavenumber.

We also observe that Tong et al. (2017), p. 765, observe that "*the modulus of porous materials is experimentally known to be a function of the strain amplitude ... the variation in modulus is negligible ... the linear elastic parameters are assumed to be constant for the subsequent discussion.*" We follow Tong et al. (2017) and treat the coefficients in equations (1) and (2) as constants.

Equation (9) yields the wavespeed as

$$V^2 = \frac{\varepsilon \pm \sqrt{\varepsilon^2 - 4\theta\psi}}{2\theta} \tag{10}$$

where

$$\begin{aligned}
&\varepsilon = \rho M + \lambda m' + 2\mu m' + m'\alpha^2 M - 2\rho_f\alpha M, \\
&\theta = \rho m' - \rho_f^2, \\
&\psi = (\lambda + 2\mu)M.
\end{aligned}$$

Provided $\varepsilon - 4\theta\psi > 0$ we see that (10) leads to a fast wave and a slow wave moving in the $\pm x$ directions with wavespeeds given by (10) as V_1, V_2, with $V_2 < V_1$.

We remark that the expression for (10) holds for V_1 since once the fast wave has passed the body ahead of the wave will no longer be in equilibrium and then V_2 will need to be calculated from (7).

To calculate V_1 in a practical case we consider an acceleration wave moving into water saturated sand, and so we are practically considering a granular material. Numerical values for the parameters in this case are given in table 1 of Tong et al. (2017), p. 764. They give values of $\rho = 2650$ kg m^{-3}, $M = 4.57$ GPa, $\lambda = 0.1$ GPa, $\mu = 0.167$ GPa, $\alpha = 0.994$, $\rho_f = 1000$ kg m^{-3}, and they show, p. 758, after equation (6), $m' = \alpha\rho_f/\beta$ where β is the porosity. For a porosity $\beta = 0.4$ as in Tong et al. (2017) this gives $m' = 2485$ kg m^{-3}. Upon employing these values in (10) we find $V_1 = 1613$ m s^{-1} which is very close to the experimentally calculated value of $V = 1626$ m s^{-1} reported by Mauger et al. (2000).

4 Wave amplitude behaviour

To find the amplitude behaviour we differentiate (1) and (2) with respect to x and take the jumps across $\mathscr{S}$. By using the Hadamard relation one may derive the equations

$$\begin{aligned}\frac{2\rho}{V}\frac{\delta A}{\delta t} + \frac{2\rho_f}{V}\frac{\delta B}{\delta t} &= [u_{xxx}](\rho V^2 - a_{11}) \\ &+ [w_{xxx}](\rho_f V^2 - a_{12}) \\ &+ \frac{c_{11}}{V^4}A^2 + \frac{2c_{12}}{V^4}AB + \frac{d_{12}}{V^4}B^2,\end{aligned} \tag{11}$$

and

$$\begin{aligned}\frac{2\rho_f}{V}\frac{\delta A}{\delta t} + \frac{2m'}{V}\frac{\delta B}{\delta t} &= [u_{xxx}](\rho_f V^2 - a_{12}) \\ &+ [w_{xxx}](mV^2 - a_{22}) \\ &- \frac{\eta}{\kappa V}B + \frac{c_{21}}{V^4}A^2 + \frac{2c_{22}}{V^4}AB + \frac{d_{22}}{V^4}B^2.\end{aligned} \tag{12}$$

We now form $(11) + \xi(12)$ and use the fact from (7) that $(a_{11} - \rho V^2)A = (\rho_f V^2 - a_{12})B$. We select $\xi = (\rho V^2 - a_{11})/(a_{12} - \rho_f V^2)$, i.e. $B = A\xi$, then using the wavespeed equation (9) the terms in $[u_{xxx}]$ and $[w_{xxx}]$ drop out of the equation $(11) + \xi(12)$.

After some calculation one may then show the equation $(11) + \xi(12)$ leads to the amplitude A satisfying the equation

$$\frac{\delta A}{\delta t} + \zeta_1 A - \zeta_2 A^2 = 0, \tag{13}$$

where

$$\zeta_1 = \frac{\eta\xi^2}{2\kappa V(\rho + 2\xi\rho_f + m\xi^2)}$$

and

$$\zeta_2 = \frac{c_{11} + c_{21}\xi + 2\xi c_{12} + 2\xi^2 c_{22} + \xi^2 d_{12} + \xi^3 d_{22}}{2V^3(\rho + 2\xi\rho_f + m\xi^2)}.$$

The solution to the Bernoulli equation (13) is

$$A(t) = \frac{A(0)}{e^{\zeta_1 t} + \dfrac{\zeta_2}{\zeta_1}(1 - e^{\zeta_1 t})A(0)}.$$

This expression represents the solution to the amplitude when dissipation is present. Additionally, this leads to blow-up in finite time of $A(t)$ when $\zeta_2 A(0) > 0$. The blow-up time is given by

$$T_D = \frac{1}{\zeta_1} \log\left(\frac{\zeta_2 A(0)/\zeta_1}{\{\zeta_2 A(0)/\zeta_1\} - 1}\right). \tag{14}$$

If we discard dissipation then $\zeta_1 = 0$ and equation (13) has solution

$$A(t) = \frac{A(0)}{1 - \zeta_2 t A(0)}.$$

If $\zeta_2 A(0) > 0$ then we find $A(t)$ blows-up in a finite time

$$T_{ND} = \frac{1}{\zeta_2 A(0)}. \tag{15}$$

If we take the limit $\zeta_1 \to 0$ then $T_D \to T_{ND}$ as it should.

5 Numerical results and conclusions

To make use of the expressions for $A(t)$ we need values for ζ_1 and ζ_2 in (13). Finding practical values for the coefficients ζ_1 and ζ_2 is not easy. Tong et al. (2017), p. 763, remark, ..."*As there are no measurements attempted on the nonlinear elastic coefficients*" and on p. 764, ... "*no measurements of coupled nonlinear elastic constants have been reported*". Fortunately, for water saturated sand Tong et al. (2017) do determine values which lead to the coefficients ζ_1 and ζ_2. To do this we firstly infer from Tong et al. (2017) that

$$\begin{aligned}
c_{11} &= -3[(\lambda_c + 2\mu) + 2(\ell + 2m)] \\
c_{12} &= c_{21} = 2\gamma_2 - \alpha M \\
d_{12} &= c_{22} = -2\gamma_3 \\
d_{22} &= 6\gamma_1,
\end{aligned}$$

where $\lambda_c = \lambda + \alpha^2 M$ (in agreement with Biot (1962); note that Donskoy et al. (1997) have a different interpretation for λ_c), and $\ell, m, \gamma_1, \gamma_2$ and γ_3 are nonlinear parameters determined by Tong et al. (2017). In fact, for water saturated sand Tong et al. (2017) find $\ell = m = -4.95$ TPa, $\gamma_1 = 1.93$ GPa, $\gamma_2 = -25.6$ TPa and $\gamma_3 = -5.9$ GPa.

Using the above values we calculate for water saturated sand, $c_{11} = 2.9685 \times 10^{13}$ Pa, $c_{12} = c_{21} = -5.1195 \times 10^{13}$ Pa, $d_{12} = c_{22} = 11.8 \times 10^9$ Pa, $d_{22} = 1.158 \times 10^{10}$ Pa, and $\xi = 1.4617$, and these lead to values of

$$\zeta_1 = 9.8164 \times 10^2 \text{ s}^{-1}, \qquad \zeta_2 = -5.70998 \text{ m s}^{-3}.$$

Since $\zeta_2 < 0$ we experience blow-up of $A(t)$ when $A(0) < 0$. To understand this recall

$$A(t) = [u_{tt}] = V^2[u_{xx}]$$

and since $u_{xx}^+ = 0$, $A(t) = V^2 u_{xx}^-$. Thus $u_{xx}^-(0) < 0$ and so when the wave amplitude blows up we see that $u_{xx}^-(t) \to -\infty$ which is consistent with u_x^- experiencing a discontinuity and a shock wave forming.

From the values above we see that if

$$|u^-_{xx}(0)| > 6.593 \times 10^{-5} \text{ m}^{-1} \tag{16}$$

when damping is present then $u^-_{xx}(t)$ blows up in a finite time. Since the right hand side of (16) is small we expect that a shock will form in this situation.

We finally compare the blow-up times T_D and T_{ND} in one case, namely when $\zeta_2 A(0) = 10^3$. Then we find

$$T_{ND} = 10^{-3} \text{ s} \qquad \text{whereas} \qquad T_D = 1.7686 \times 10^{-3} \text{ s}.$$

This shows damping does slow up the amplitude blow - up as is to be expected.

We have developed an exact acceleration wave analysis for the equations for nonlinear wave motion in a porous body given by Donskoy et al. (1997). These equations are derived from a strain energy function which depends upon invariants I_1, I_2 and I_3 of the strain tensor, see Donskoy et al. (1997), equation (8). As Donskoy et al. (1997) remark, the coefficients of their equations characterize the nonlinear behaviour of an *isotropic* porous medium. It is shown that for certain parameter ranges there are two waves, a fast one and a slow one. The amplitude of the leading wave is calculated exactly including when dissipation is specifically present. Under appropriate initial conditions it is also shown that the amplitude may blow-up in a finite time. This is believed to be associated with the development of a shock wave in the porous body.

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